Michael J. Ruiz, "The Teleconverter Problem," *American Journal of Physics* **49**, 1006, 1054 (November 1981). The posed problem is on page 1006; the solution is on page 1054.

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Charron ring. However, if in such cases the ellipse is narrow and the amplitude constant, the intrinsic precession might still be expected to be *proportional* to the minor diameter of the ellipse. Conceivably, a lucky combination of values of the several parameters could give zero intrinsic precession. The problem does not invite analytical treatment, because the wire slides on the ring during part of the time of contact. A further qualification should be made, and that concerns the limiting case as the minor diameter of the ellipse approaches zero. The derivation applies to a pendulum on a nonrotating planet. On the rotating Earth, the path cannot become a straight line in the limit. Consider for simplicity a pendulum at a geographic pole. If the bob comes to rest in the local frame at the extremity of the swing, which would happen under the constraint ideally exerted by a Charron ring, its path in that frame is a figure of n cusps. In the cosmic inertial frame it is an ellipse. The ellipse, of readily calculable area gives, by the intrinsic precession formula, an angular velocity of the $3\omega_e a^2/8l^2$, where a and l are the amplitude and the length of the pendulum and ω_e is the angular velocity of the Earth. This is to be subtracted from the expected Foucault turning rate. Typically, the correction is less than $\frac{1}{2}$%. If, on the other hand, the path in the cosmic frame is a straight line, it will be seen in the local frame as a rosette. The cusp and rosette paths have been treated at length by W. S. Kimball, Am. J. Phys. **13**, 271–75 (1945); and by W. B. Somerville, Quart. J. R. Astron. Soc. **13**, 40-62 (1972).

[4]This is strikingly demonstrated if the suspension is in the shape of a *Y*, attached to the ceiling at two points, and if the lengths are such that the periods in the planes parallel and perpendicular to the *Y* are different by, say, 5%. For maximum effect the pendulum should be launched at 45 deg to the plane of the *Y*. The sense (clockwise or counterclockwise) in which ellipticity grows alternates from quadrant to quadrant in which the motion is started. A photograph of such motion [H. R. Crane, Phys. Teach. **8**, 182 (1970)] exhibits the constant rate of growth of the minor diameter at the beginning. The path starts near the center of the photograph and is counterclockwise. The intrinsic precession, which is a smaller effect, can be seen as a rotation of the principal axes of the ellipses, between the start and the finish.

[5]If, in the 2-m pendulum used as an example, the periods in the two perpendicular directions, each at 45 deg to the direction of swing, were to differ by only a part in 10^5 (the effective lengths differing by 0.04 mm) the minor diameter would grow to 2.2 mm, enough to cancel the turning due to the Earth's rotation, in less than 200 cycles.

[6]F. Charron, Bul. Soc. Astron. France **45**, 457 (1931).

[7]Monash University in Australia [C. F. Moppert and W. J. Bonwich, Quart. J. R. Astron. Soc. **21**, 108–18 (1980)]. University of Hawaii (private communication).

[8]The problem is related to that which Christian Huygens addressed in 1658 in relation to the plane (clock) pendulum, with the object of making the period independent of amplitude. Instead of reducing the restoring force in the middle region as is done here, his method was to increase it in the outer regions. For a discussion see A. L. Rawlings, *The Science of Clocks and Watches* (Caldwell, Luling, TX, 1974).

[9]The elliptical motion is only lightly restrained for a reason. It is better to allow a small amount of residual ellipticity, whose effect is nullified by the fixed magnet, than to use a strong restraint, which may modify the turning rate. The restraint is furnished by a light weight (15 g) annular ring, (loose) located below the bob. The stem below the bob pushes it one way and then the other, at the extremities of the swing, about 1 mm. That is sufficient restraint, in view of the facts (i) that a little ellipticity is harmless, and (ii) that the supporting gimbal contains a screw by which the relative heights of the two axes of rotation is adjusted to minimize ellipticity at its source. The small energy loss at each push of the ring very effectively keeps the amplitude constant to a fraction of a millimeter. In a test in which the compensating magnet was removed, the ellipticity of about 2 mm minor diameter was maintained, the intrinsic precession agreed with the formula to within about 20%, and that was mainly the uncertainty in maintaining the minor diameter. Thus under the above set of conditions, the pendulum can be said to be nearly free.

[10]Listed in the catalog of Edmund Scientific Co., 7785 Edscorp Bldg., Barrington, NJ 08007.

PROBLEM

A primary camera lens (focal length is typically $f_1 = 50$ mm) can be converted into a telephoto lens (a lens with a longer focal length, e.g., 100 or 150 mm) by placing a secondary lens (teleconverter) between the primary lens and the film plane as indicated in Fig. 1.

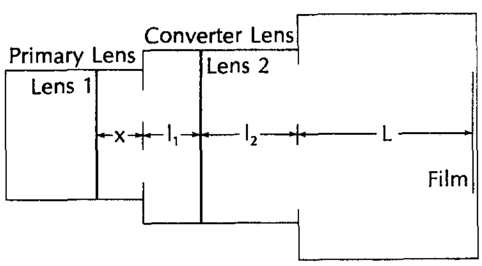

Fig. 1. Optical arrangement for teleconverter lens, primary lens, and camera body.

(a) Show that the multiplication factor α (defined as the ratio of the effective focal length of the lens system to the focal length of the primary lens) is $\alpha = (L + l_2)/(L - l_1)$. Assume that the lenses are thin.

(b) Show that the focal length of the diverging converter lens is $f_2 = -(L - l_1)(L + l_2)/(l_1 + l_2)$.

(Solution is on page 1054.)

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It is also the theory that is obtained by asking for the classical field corresponding to quantum-mechanical particles of zero rest mass and spin two. The theory was obtained by Fierz and Pauli[5] in this way in 1939. This theory predicts a precession greater than that predicted by general relativity by a factor of 4/3.

If our imaginary physicist limited his considerations to tensor theories of rank less than three, as we have done, he could have eliminated all theories except Whitehead's because of conflict with observations of the precession of planetary orbits. He might then investigate other small effects predicted by the theory. Curiously, Whitehead's theory makes the same predictions as does general relativity for the other two classical tests of general relativity, the deflection of light by the sun and the gravitational redshift.[6]

Our imaginary physicist would feel strongly compelled to accept as the true theory of gravitation the theory of Whitehead—a theory that we know today to be wrong. After the publication of Whitehead's theory, almost half a century was to elapse before Will[7] showed that this theory is in conflict with experimental observations. Whitehead's theory predicts Earth tides with a 12-hr sidereal period due to the mass of our galaxy. The amplitude of these tides is bigger by a factor of about 200 than the upper limit of sensitive gravimeter measurements.

In the actual sequence of historical events, Einstein's theory appeared first and was well established in the thinking of the scientific world before Whitehead's theory was published. It was probably partly due to this and partly due to the great philosophical and aesthetic appeal of general relativity that Whitehead's theory never became a strong competitor to Einstein's in spite of its greater simplicity.

[1]For a discussion of alternative theories of gravitation and the comparison of their predictions with general relativity and with observations, see the review by G. J. Whitrow and G. E. Morduch in *Vistas in Astronomy*, edited by Arthur Beer (Pergamon, New York, 1965), Vol. 6, p. 1–67. An interesting review of Lorentz-covariant scalar theories of gravitation is that of A. L. Harvey, Am. J. Phys. **33**, 449 (1965).

[2]J. A. Wheeler and R. F. Feynman, Rev. Mod. Phys. **17**, 157 (1945); **21**, 425 (1949).

[3]A. N. Whitehead, *The Principle of Relativity* (Cambridge University, Cambridge, 1922). More recent papers on Whitehead's theory are J. L. Synge, Institute of Fluid Dynamics and Applied Mathematics, Univ. Maryland, Lecture Series 5, 1951 (unpublished); J. L. Synge, Proc. Soc. London A **211**, 303 (1952); C. B. Rayner, *ibid.* **222**, 509 (1954); A. Schild, *ibid.* **235**, 202 (1956).

[4]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1970), Chap. 18.

[5]M. Fierz and W. Pauli, Proc. Soc. London A **173**, 211 (1939).

[6]J. L. Synge, Ref. 3.

[7]C. M. Will, Astrophys. J. **169**, 141 (1971).

SOLUTION TO THE PROBLEM ON PAGE 1006

The magnification for a two-lens combination (thin lenses) is[1]

$$M = f_1 s_i / [d(s_0 - f_1) - s_0 f_1], \qquad (1)$$

where d is the distance between the lenses $(x + l_1)$, s_0 is the distance from the object to the primary lens (which is large in telephoto photography), and s_i is the distance from the converter lens to the image $(L + l_2)$. For distant objects s_i is essentially the back focal length (bfl) of the lens system: bfl $= f_2(f_1 - d)/(f_1 + f_2 - d)$.[1] Substituting this relation for s_i into (1) and expanding in powers of f_1/s_0 (a very small quantity), the first-order term is

$$M = \left(\frac{f_1}{s_0}\right)\left(\frac{-f_2}{f_1 + f_2 - d}\right). \qquad (2)$$

From (2) it is apparent that $\alpha = f_2/(f_1 + f_2 - d)$. Since $f_1 - d = (x + L) - (x + l_1) = L - l_1$ and bfl $= \alpha(f_1 - d) = \alpha(L - l_1)$, which is also equal to $L + l_2$, we readily obtain $\alpha = (L + l_2)/(L - l_1)$. After a little algebra, using the above relations, we find $f_2 = -(L - l_1)(L + l_2)/(l_1 + l_2)$. In Table I a comparison is made between calculated values for α, using the thin-lens approximation and manufacturer values for five commercial teleconverters.

[1]E. Hecht and A. Zajac, *Optics* (Addison-Wesley, Reading, MA, 1974).

Table I. Comparison between values for multiplication factor (α) of teleconverter lenses and calculated values from converter and camera parameters.

Camera	Teleconverter (α)	l_1 (mm)	l_2 (mm)	Calculated (α)
Canon AE-1 ($L = 40$ mm)	Soligor (3)	16	29	2.9
Nikon F ($L = 46$ mm)	Kenko (2)	23	4	2.2
Nikon F ($L = 46$ mm)	Komura (2)	24	2	2.2
Pentax K ($L = 45$ mm)	Komura (2)	15	15	2.0
Pentax K ($L = 45$ mm)	Soligor (3)	22.5	22.5	3.0