Michael J. Ruiz, "Fine Structure of Quarkonium in the Covariant Harmonic-Oscillator Quark Model," *Physical Review D* **34**, 1640-1641 (September 1986).

Fine structure of quarkonium in the covariant harmonic-oscillator quark model

Michael J. Ruiz

Department of Physics, University of North Carolina at Asheville, Asheville, North Carolina 28804

(Received 28 March 1986)

A general formula relating the $^3P_2$-$^3P_1$ and $^3P_1$-$^3P_0$ fine-structure splittings in quarkonium is derived within the context of the covariant harmonic-oscillator quark model. The unique feature of the formula is its ability to predict the fine-structure splittings for $u\bar{u}(d\bar{d})$, $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ with only one essentially free parameter. Overall agreement with experiment is reasonably good.

The relativistic harmonic-oscillator model[1] is based on a mass-squared operator, where internal quark motion is described by a four-dimensional oscillator potential. The unperturbed square of the mass, $M^2 = A + N\omega$, rises linearly with the internal quantum number ($N = 0,1, 2,3,\ldots$), where the Regge slope determines the binding spring constant ω. Normalizable covariant-oscillator wave functions have been constructed, restricting timelike oscillations to a ground-state Gaussian in the relative-time coordinate;[2] and covariant oscillators have been applied to nucleon electromagnetic form factors,[3,4] $-g_A/g_V$,[5] hadron decays,[6,7] and other processes.[8]

Shortly after the appearance of the chromodynamic one-gluon-exchange mechanism of De Rújula, Georgi, and Glashow,[9] Kim proposed a covariant one-gluon-exchange formalism[10] using relativistic-oscillator wave functions. The covariant model[10] employs a real gluon propagator[11] formed from a symmetrization of retarded and advanced Green's functions. Quark spin is incorporated through projection operators.[6,10] A relativistic Fermi-Breit formula is produced without making the usual static nonrelativistic approximation, and an encouraging result is found relating the meson and baryon nonstrange hyperfine splittings.[12]

Interest in P-wave meson mass spectra was heightened during the early work on charmonium. Schnitzer[13] demonstrated that a nonrelativistic potential having a Coulomb and linear component with a Lorentz-vector character bounds the ratio of the $^3P_2$-$^3P_1$ and $^3P_1$-$^3P_0$ splittings: $0.8 \le R \le 1.4$, where

$$R = [M(^3P_2) - M(^3P_1)]/[M(^3P_1) - M(^3P_0)] . \quad (1)$$

The lower bound corresponds to a purely Coulombic interaction, while the upper bound represents a completely linear potential. The range of R does not include the experimental value for charmonium, $R_c = 0.48 \pm 0.01$.[14] A general power law $V(r) = Ar^n$ ($n \ge -1$, $n \ne 0$) with a Lorentz-vector structure[15] likewise does not fit the data since the calculated value is $R = [2(13+n)]/[5(5-n)]$.

In order to obtain lower values of R to fit the charmonium spectrum, a Lorentz-scalar feature was introduced[16] and a quark-gluon anomalous color moment considered.[17] Subsequently, a variety of models[18] has appeared incorporating scalar confining terms. Other approaches[19,20] have also met with a degree of success in fitting quarkonium spectra. However, these models either focus on heavy quarkonium systems or introduce several free parameters in attempting to produce a more complete spectroscopy that fits experimental data.

The covariant harmonic-oscillator model presents a unified picture of both heavy- and light-quarkonium fine structure with one free parameter: the relative mixing of Lorentz-scalar and -vector structures. This mixing parameter can be determined from the ratio of the $^3P_1$-$^3P_0$ and $^3S_1$-$^1S_0$ splittings of charmonium. The fine-structure ratios can then be calculated for $n\bar{n}$, $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$, where n refers to the nonstrange quarks u and d. Calculating fine-structure ratios eliminates the introduction of the quark-gluon coupling strength and any associated phenomenological dependence on quarkonium mass.

The details of the covariant harmonic-oscillator formalism can be found in Ref. 10. The calculation of R is lengthy, but straightforward. The formula for R, where now the squares of the meson masses are to be used in Eq. (1), is

$$R = \frac{2(13/5 - k) - (9/40 + 13k/40)x}{5 - k + (33/16 + k/16)x} , \quad (2)$$

where k is the relative amount of scalar structure present, and $x = \omega/u^2$ with $u = m_q + M/2$, m_q being the quark mass and M the unperturbed meson mass.

The parameter k is chosen by considering the experimental ratio

$$[M^2(^3P_1) - M^2(^3P_0)]/[M^2(^3S_1) - M^2(^1S_0)]$$

for charmonium, which is 0.9.[14] The leading theoretical term contributing to this ratio is $(5-k)/4$, which indicates that k is roughly 1.5, the value taken for the following analysis. The unperturbed meson mass M is obtained from the spin-averaged experimental masses, whenever possible. For example, the $n\bar{n}$ system ($I = 1$) has four P states: $A_2(1320)$, $A_1(1270)$, $\delta(980)$, and $B(1235)$, corresponding, respectively, to $^3P_2$, $^3P_1$, $^3P_0$, and $^1P_1$.[14] In the covariant-oscillator model, the square of the meson mass is given as a sum of an unperturbed part M^2 and a perturbation $\delta M^2 = a \mathbf{S}_1 \cdot \mathbf{S}_2 + b \mathbf{L} \cdot \mathbf{S} + cT$, where T is the tensor term. The four experimental masses can be used to eliminate a, b, and c, giving the experimental value $M = 1.261$ GeV. The quark masses have their standard values for constituent quarks. Values for the Regge slope parameter ω are taken from the analysis of Chang and Nelson,[21] who

TABLE I. Quarkonium systems and fine-structure ratios R.

System	M (GeV)	m_q (GeV)	ω (GeV^2)	x	R_{th}	R_{expt}
$n\bar{n}$ ($I=1$)	1.261[a]	0.336[b]	1.11[c]	1.188	0.22	0.20[a]
$s\bar{s}$ ($I=0$)	1.31[d]	0.520[e]	1.165[c]	0.844	0.30	0.29[a]
$c\bar{c}$ ($I=0$)	3.5[f]	1.5[g]	2.0[c]	0.189	0.53	0.48[a]
$b\bar{b}$ ($I=0$)	9.9[f]	5.0[g]	5.82[c]	0.0588	0.60	0.68[h]
$t\bar{t}$ ($I=0$)				~ 0	0.63	

[a]Particle Data Group, Ref. 14, using the squares of the nominal masses.
[b]From the standard proton-magnetic-moment analysis, see Ref. 9.
[c]From Chang and Nelson, Ref. 21.
[d]The average of the masses $f'(1525)$, $E(1420)$, and $S^*(975)$, which are taken to be the states $^3P_2$, $^3P_1$, and $^3P_0$, respectively.
[e]From the analysis found in Ref. 9, using, however, the squares of the hadron masses.
[f]Experimental average of the triplet.
[g]The nominal value.
[h]From R. Nernst et al. (Crystal Ball), Phys. Rev. Lett. **54**, 2195 (1985); H. Albrecht et al. (ARGUS), Phys. Lett. **160B**, 331 (1985).

use a relativistic-oscillator model to fit the empirical Regge trajectories with a unified formula. The parameter x is then determined, allowing for a numerical evaluation of R.

The results for both light and heavy quarkonia are listed in Table I. The overall comparison with experiment is reasonably good. The lighter quarks have more internal quark motion, which is reflected in a higher value for x. The higher values for x result in lower values for R, as evident from Eq. (2) and Table I. The heavier-quark systems undergo less internal motion (x relatively small), and therefore have higher values for R. For extremely non-relativistic quarks ($x=0$) the value for R reaches its maximum value.

[1]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3**, 2706 (1971).

[2]Y. S. Kim and M. E. Noz, Phys. Rev. D **8**, 3521 (1973); M. J. Ruiz, ibid. **10**, 4306 (1974).

[3]K. Fujimura, T. Kobayashi, and M. Namiki, Prog. Theor. Phys. **43**, 73 (1970); **44**, 193 (1970).

[4]R. G. Lipes, Phys. Rev. D **5**, 2849 (1972); see S. Ishida, K. Takeuchi, S. Tsuruta, M. Watanabe, and M. Oda, ibid. **20**, 2906 (1979), for a review of electromagnetic-form-factor calculations in the covariant harmonic-oscillator model; M. L. Haberman, ibid. **29**, 1412 (1984).

[5]M. J. Ruiz, Phys. Rev. D **12**, 2922 (1975).

[6]Y. S. Kim and M. E. Noz, Phys. Rev. D **12**, 129 (1975); Prog. Theor. Phys. **57**, 1373 (1977).

[7]I. Montvay and J. Spitzer, Phys. Rev. D **18**, 2503 (1978); L. Maharana and S. P. Misra, ibid. **18**, 2530 (1978).

[8]P. E. Hussar, Phys. Rev. D **23**, 2781 (1981); for a list of references dealing with partons, jet phenomena, and mathematical discussions of the covariant harmonic oscillator, see D. Han, M. E. Noz, Y. S. Kim, and D. Son, ibid. **27**, 3032 (1983).

[9]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).

[10]Y. S. Kim, Phys. Rev. D **14**, 273 (1976).

[11]For similar models, see S. Blaha, Phys. Rev. D **10**, 4268 (1974); S. Ishida, S. Hinata, M. Oda, K. Takeuchi, and K. Yamada, Prog. Theor. Phys. **71**, 806 (1984).

[12]M. J. Ruiz, Phys. Rev. D **30**, 683 (1984).

[13]H. J. Schnitzer, Phys. Rev. Lett. **35**, 1540 (1975).

[14]Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984).

[15]J. Pumplin, W. Repko, and A. Sato, Phys. Rev. Lett. **35**, 1538 (1975).

[16]A. B. Henriques, B. H. Kellett, and R. G. Moorhouse, Phys. Lett. **64B**, 85 (1976).

[17]H. J. Schnitzer, Phys. Lett. **65B**, 239 (1976); C. E. Carlson and F. Gross, ibid. **74B**, 404 (1978).

[18]A representative list includes H. J. Schnitzer, Phys. Rev. D **18**, 3482 (1978); D. Beavis, S.-Y. Chu, B. R. Desai, and P. Kaus, ibid. **20**, 743 (1979); W. Buchmüller, Phys. Lett. **112B**, 479 (1982); S. N. Gupta, S. F. Radford, and W. W. Repko, Phys. Rev. D **26**, 3305 (1982); **31**, 160 (1985); **34**, 201 (1986); R. McClary and N. Byers, ibid. **28**, 1692 (1983); J. Zuk, G. C. Joshi, and J. W. G. Wignall, ibid. **28**, 1706 (1983); M. Bander, D. Silverman, B. Klima, and U. Maor, ibid. **29**, 2038 (1984); H. Grotch, D. A. Owen, and K. J. Sebastian, ibid. **30**, 1924 (1984).

[19]A representative list includes J. S. Kang, Phys. Rev. D **20**, 2978 (1979); D. P. Stanley and D. Robson, ibid. **21**, 3180 (1980); W. Buchmüller, G. Grunberg, and S.-H. H. Tye, Phys. Rev. Lett. **45**, 103 (1980); **45**, 587(E) (1980); W. Buchmüller and S.-H. H. Tye, Phys. Rev. D **24**, 132 (1981); E. Eichten and F. Feinberg, ibid. **23**, 2724 (1981); J. Baake, Y. Igarashi, and G. Kasperidus, Z. Phys. C **9**, 203 (1981); A. Khare, Phys. Lett. **98B**, 385 (1981); P. Moxhay and J. L. Rosner, Phys. Rev. D **28**, 1132 (1983); M. Arafah, R. Bhandari, and B. Ram, Lett. Nuovo Cimento **38**, 305 (1983); N. Mukerji and C. S. Kalman, ibid. **41**, 513 (1984); D. Gromes, Z. Phys. C **26**, 401 (1984); K. Heikkilä, N. A. Törnqvist, and S. Ono, Phys. Rev. D **29**, 110 (1984); **29**, 2136(E) (1984).

[20]J. Pantaleone, S.-H. H. Tye, and Y. J. Ng, Phys. Rev. D **33**, 777 (1986), and references therein.

[21]N.-P. Chang and C. A. Nelson, Phys. Rev. D **19**, 3336 (1979).