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Road mirage angle

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Abstract

A video is taken of a road-sign mirage from the passenger seat in a car traveling at constant speed on a highway. The video spans the duration of seeing the mirage of the sign, viewing the vanishing of the mirage as the car approaches, and passing the road sign. The mirage angle, defined as the angle with respect to the horizontal at the moment the observer notes the vanishing of the mirage, can be determined from the video, the speed of the car, and known dimensions of the standard road sign. The value can be checked with a theoretical formula using the ambient weather temperament and consulting references to determine the temperature of the air in contact with the road surface. Agreement between observation and theory is within the error bounds. In the conclusion, a quick observational estimate is made in the car without needing the video.

Background

When I was 10 years old in 1961 my family made a trip by car from our home in Camden, New Jersey, USA to Beaufort, North Carolina, USA. During the trip, I saw what appeared to be water on the road as my dad drove the family car. When the car approached the water, the water vanished, but only to reappear still farther ahead. I asked my dad 'Why is there water on the road that disappears?' He said, 'Son, that is a mirage.'

Mirages are very common and discussed in classic texts by Minnaert [1] and Greenler [2]. An excellent resource on mirages appearing in this journal is the featured article by Vollmer [3]. The road

mirage is classified as an inferior mirage since the image appears below the object, which can be a sign, other cars, or the sky. The cause is the temperature gradient just above the surface of the hot road. The air becomes hotter and less dense as the road surface is approached from above, resulting in a smaller index of refraction near the pavement [1]. Such an inferior mirage also appears in a hot desert, where the illusion of water gives the desert traveler false hope of getting a drink.

Over a body of water the temperature gradient can be reversed, a phenomenon called a temperature inversion. When this occurs, a superior mirage appears above the object. A classic case over the English Channel has been described by Minnaert [1]: 'observations of magnificent "superior" mirages were carried out from the south coast of England by looking through a telescope across the Channel, sometimes in the evening after a very hot day, sometimes while a mist was lifting.' [1]

Superior mirages include towering castles named after Morgan le Fay, who 'according to Celtic legend and Arthurian romance, was a fairy, half-sister of King Arthur, who exhibited her powers by the mirage.' [4]. This mirage of 'marvelous crystal palaces' is called Fata Morgana, 'Italian for Morgan le Fay, or Morgan the fairy,' [4] as it was observed over the Strait of Messina in Italy [1]. Ahrens and Henson [5] describe the temperature profile for the Fata Morgana in the following way: 'The Fata Morgana is observed where the air temperature increases with height above the surface, slowly at first, then more rapidly, then slowly again.'

Observing the road mirage

In June 2019, I decided to finally revisit Beaufort for the first time since I was a child, this time traveling with my wife from our home in Asheville, North Carolina. As we neared Beaufort, driving on a hot dry highway near New Bern, I saw striking mirages of orange road signs, as shown in figure 1. I began shooting a video after it occurred to me that the mirage angle, defined as the observer angle with the respect to the horizontal at the moment the mirage vanishes, can be determined from a video and the constant speed of the car. Knowing the outside ambient weather temperature (temperature at the height of the observation) and the temperature of the layer of air just above the road (to be determined), the mirage angle can also be calculated from theory. Calculations of the mirage angle from both observation and theory follow in the next sections.



Figure 1. Road mirage which includes two orange signs with a white vehicle between them. Note the familiar mirage appearance of water on this dry hot day. The image is a detail of a frame from the author's video accompanying this paper [6].

Mirage angle from observational data

A link to my video, the main source for the observational data, is included [6]. The exact moment the

video starts is not important. However, it is necessary to keep the camera rolling through the

vanishing of the mirage and until the sign is passed by the car. The car should be traveling at a fairly

constant speed, which is easy to achieve on an uncrowded highway. The left orange sign in the video

is shown in figure 2, a detail from a screenshot just before passing the sign and about 16 seconds

after the mirage vanished. Each side of the standard 'Low/Soft Shoulder' diamond-shaped sign for a highway in the United States is 3 ft (0.91 m) [7]. The side of the sign serves as the calibration to obtain the distance from the bottom of the sign to the ground, a distance found to be 1.8 m. The bottom of the sign is important because when the orange of the sign mirage vanishes, the last rays reaching the observer come from the bottom of the sign (least overall refraction of the light). To find the 1.8 m, first note that the vertical diagonal of the sign is $0.91 \text{ m} \sqrt{2} = 1.3 \text{ m}$, the hypotenuse of a right triangle with a pair of 0.91 m sides. The 1.8 m is found from the ratio of the sign's diagonal to the vertical length of the support for the sign.



Figure 2. Calibration of the distance from the bottom of the sign to the ground as the standard 'Low/Soft Shoulder' sign is 3 ft (0.91 m) on each side.

The height of the camera in the car from the surface of the road can be measured directly

with a meter stick when the car is safely parked. The result was 1.2 m. The car was traveling

70.0 mi \cdot h⁻¹ = 31.3 m \cdot s⁻¹ and the time from the vanishing of the mirage to reaching the sign was 16 seconds. Therefore the distance from the point where the mirage was last seen to the sign is 31.3 m \cdot s⁻¹ \cdot 16 s = 501 m, shown in figure 3, where the mirage angle is α . The lowest point of the light path in figure 3 is not centered since the vertical height of the observer (1.2 m) and height of the sign post (1.8 m) are not equal. Note that the light rays are straight lines except for the very short segment close to the road. Before calculating the mirage angle it is important to discuss this feature since often the curvature of the refracted light path is highly exaggerated in many qualitative treatments of the inferior mirage for illustration purposes.



Figure 3. Diagram for the position where the mirage of the orange is last seen. The curved segment is a very small portion, most of it being within 1 cm of the road surface and less than 1% of the total horizontal length of the light path.

The light path consists of two long straight line segments because the temperature in the air is constant until the ray is very close to the road surface. Minnaert [1] points out that the air temperature above a hot road drops by 20 °C to 30 °C in the first centimeter above the surface [8]. Students can verify that air temperature is not perceived to change until one is almost touching the

asphalt, where care should be taken because the asphalt can cause a burn [9]. Above 1 cm, the lapserate is 'of the order of $1^{\circ}-2^{\circ}$ C. per cm. at higher levels near a heated surface' [8]. Minnaert describes the refraction shown in figure 3 as 'a rather sudden deflection' [1].

For a pavement temperature of 57 °C and weather temperature of 30 °C [9], a 25 °C drop in the first cm (average of Minnaert's 20 °C and 30 °C) brings the temperature at 1 cm to almost the ambient weather temperature. An estimate of the horizontal extent of the curved portion in figure 3 on either side of the minimum point is $\frac{0.01 \text{ m}}{1.80 \text{ m}} \cdot 301 \text{ m} = 1.7 \text{ m}$, compared to the hundreds of metres of the entire horizontal length. Therefore the mirage angle can be calculated from the right triangle in figure 3 as $\alpha = \tan^{-1} \left[\frac{1.8}{301} \right] = 0.34^{\circ}$. This angle is very small due to the large horizontal distances of the triangles in figure 3. Next we consider the uncertainty.

First, the road distance of 301 m in figure 3 is not a direct line from the car to the sign. The car was in the right lane and the sign was located to the left of the left lane. Since the highway lane is the standard (USA) 3.66 m (12 ft), the sign was at least 5.5 m (a lane and a half) from the car when it passed the sign. To be very conservative, consider the distance to the sign as the car passes it to be 10 m. Then, using the Pythagorean Theorem with a positional shift of 10 m, gives a hypotenuse $h = \sqrt{301^2 + 10^2} = 301.17$ m, still the distance of 301 m in figure 3 to three significant figures. The new mirage angle, using 301.17 m, is $\alpha = \tan^{-1} \left[\frac{1.8}{301.17} \right] = 0.34^\circ$, the same as before.

The confidence that heights 1.8 m and 1.2 m in figure 3 are known to two significant figures is not shared for the horizontal distance measurement. The horizontal distance is subject to uncertainties in the constant car speed and timing the car from the vanishing of the mirage to passing the sign. A reasonable estimate for the speed including uncertainty is $70 \pm 1 \text{ mile} \cdot h^{-1} = 31.3 \pm 0.5 \text{ m} \cdot \text{s}^{-1}$ and for the time is $16.0 \pm 0.5 \text{ s}$. These uncertainties lead to mirage angles ranging from 0.33° to 0.36°. Therefore, the experimental mirage angle can be reported to two significant figures as

$$\alpha_{\rm exp} = 0.34^{\circ} \pm 0.02^{\circ} \tag{1}$$

Before proceeding to theory it is important to discuss a few safety issues, especially since students are involved.

Safety issues for students

There are three main safety issues to consider. First, the driver needs to concentrate on the road. The student in the passenger seat taking the video should focus on the video without distracting the driver. If the driver needs to slow down due to traffic, the video should be aborted. Wait until traffic allows for the driver to maintain a constant speed and watch for the next sign. Second, the student should not attempt to measure sign dimensions directly as a highway is not for pedestrians. The signs are standard and dimensions can be found in reference materials.

Finally, the temperature of the asphalt surface will be required later in the theory discussion. This temperature can be found in reference material to be discussed, based on the ambient weather temperature provided. The weather temperature can be found from the local radio station, the Internet, or the car if it should have an external temperature sensor reading on the dashboard. Under absolutely no circumstances should the student get out of the car and try to directly measure the highway road surface temperature with a handheld infrared thermometer. In fact, the surface temperature can vary a few °C from place to place on the road, as the author verified on his safe asphalt driveway. Therefore, published reference charts by engineers are better sources for estimating average road surface temperatures.

Road mirage theory

The road mirage is caused by refraction through a region with a gradient in the index of refraction. Warmer air closer to the road has less index of refraction compared to the cooler air above. The speed of light is greater in the less dense warmer air near the road. Figure 4 provides a discrete model with three horizontal regions, each with a lesser uniform index of refraction as the light travels downward. Snell's law gives the relationship between the angles measured with respect to the normal for each boundary. For the top boundary, $n_1 \sin \theta_1 = n_2 \sin \theta_2$ and for the lower boundary, $n_2 \sin \theta_2 = n_3 \sin \theta_3$. Note that

$$n\sin\theta = const\tag{2}$$

for the horizontal uniform layers.



Figure 4. Simple model of discrete horizontal layers of uniform index of refraction decreasing as the layers get lower and lower.

To arrive at the proper model for a mirage, the discrete uniform layers of figure 4 need to be replaced by a continuous change in index of refraction. See figure 5, where very near the road a gradient replaces the abrupt changes in the index of refraction found in figure 4. Equation (2), $n \sin \theta = const$, is still true [10, 11] since the infinitesimal layers are still parallel to each other.

Applying $n \sin \theta = const$ at the observer height where the ambient weather temperature applies (designated with the subscript cold) and at the air layer adjacent to the road surface (designated with the subscript hot) in figure 5,



Figure 5. Path for light ray where the index of refraction varies continuously when very close to the road surface. Most of the curved part of the path is less than 1 cm above the road surface.

The ambient weather temperature is designated as the cold temperature T_{cold} with the refractive index of air at this temperature represented by n_{cold} . The hot temperature T_{hot} is the temperature of the infinitesimal horizontal layer just above the road surface, where the index of refraction for the air is written as n_{hot} . Note that the temperatures of horizontal infinitesimal layers of air are equal to the ambient weather temperature T_{cold} until one reaches a few centimeters above the pavement surface.

A student may ask how the horizontal light ray at the bottom of the optical path in figure 5 can refract. A quick answer resorts to wave theory and the related wave front along the light path [12] due to the finite diameter of the light ray bundle. The top of the wave front is in a region with greater index of refraction where light travels slower. The bottom part of the wave front is in a region with less index of refraction where light travels faster. The faster speed of the bottom part of the wave front causes the wave direction to shift upward [12]. Another answer is found through a clever analysis with ray theory where the radius of curvature of the light path is calculated at the bottom in figure 5 using standard methods from calculus [13].

An equation for the mirage angle α can now be found. Since $\theta + \alpha = 90^{\circ}$ in figure 5, equation (3) becomes $n_{\text{cold}} \sin(90^{\circ} - \alpha) = n_{\text{hot}}$, which can be written as

$$\cos \alpha = \frac{n_{\text{hot}}}{n_{\text{cold}}}.$$
 (4)

The angle α is small since the mirage is observed quite a distance away. The measured angle from our car video earlier gave $\alpha = 0.34^\circ = 0.006$ radian <<1, indeed an extremely small angle. Therefore,

the cosine in equation (4) can be expanded as $\cos \alpha = 1 - \frac{\alpha^2}{2}$, keeping the first two terms of the

Maclaurin series. Using this expansion in equation (4), $1 - \frac{\alpha^2}{2} = \frac{n_{\text{hot}}}{n_{\text{cold}}}$. Solving for α , the mirage angle

$$\alpha = \sqrt{2(1 - \frac{n_{\text{hot}}}{n_{\text{cold}}})} \,. \tag{5}$$

A formula to calculate the refractive index of air will shortly be introduced. The two temperatures T_{cold} and T_{hot} will be needed. The video of the road mirage was taken 3:37 pm. The New Bern weather station at a very small airport a few kilometers away reported a temperature 28.3 °C at 2:54 pm, the maximum for the day, and 26.7 °C at 3:54 pm. New Bern is a town (population about 30,000) in a rural area without any major industries to produce urban heat islands. The nearest industrial complex is the Raleigh-Durham area about 180 km northwest of New Bern. A weather map provided by the National Weather Service (USA) for a rectangular area roughly 200 km by 150 km with Raleigh (the state capitol) near the center [14] shows temperatures varying by only ± 1 °C (± 2 °F) or less over this entire large neighboring region. Therefore, the cold temperature can be safely estimated as $T_{cold} = 27.5$ °C ± 1.0 °C, the temperature at our 1.2 m height of observation and even closer to the road until within a few centimeters above the pavement.

During a calm day, the infinitesimal layer of air in contact with the asphalt road is considered to be in thermal equilibrium with the asphalt road surface. This assumption is supported by plots of temperature profiles near hot road surfaces [15, 16] constructed from field measurements. Hui [15] used thermocouples "to measure the temperature on the payment surfaces and of near-surface air at various heights above the pavement surface." Yang et al plotted temperature versus height from a hot road surface to 7 cm above the pavement using fifteen field temperament measurements at different heights. In each case smooth curves were fitted to the data where the graphs start at the hot pavement temperature and then temperature decreases with altitude rapidly near the road. Choosing a very small height = ε above the pavement surface on these plots gives an air temperature essentially equal to the pavement temperature at height = 0. Therefore T_{hot} , the temperature of air in contact with the asphalt road surface, is taken to be the pavement temperature in our analysis.

The common asphalt highway is hot mix asphalt (HMA), the 'excellent paving material with over 100 years of proven performance' and it 'has been recorded that 90% hard-surfaced roads in the world is HMA' [17] in 2010. An excellent reference published by the US Transportation Research Board [18] can be consulted to estimate asphalt surface temperatures. This reference contains plots of maximum asphalt pavement temperature versus latitude for different maximum air temperatures.

The latitude for New Bern is 35° N and $T_{cold} = 27.5$ °C ± 1.0 °C. Our observation was made about 30 minutes after the maximum temperature 28 °C for the day. From the plots in reference 18, the road surface temperature corresponding to 27.5 °C is 53 °C at the New Bern latitude.

The authors Solaimanian and Kennedy of Ref. 18 assess the uncertainties of their theoretical model as follows.

In 83 percent of the cases, the proposed equation predicted the pavement temperature

within 3 °C, which is well within reasonable limits, considering the numerous uncertainties that exist in material properties, accuracy of measurements, variability of environmental factors (wind, sunshine, etc.), and inclination of the pavement surface in receiving radiation. [18]

Therefore, $T_{\rm hot}$ can be taken to be $T_{\rm hot}$ = 53.0 °C ± 3.0 °C.

The index of refraction for dry air in the optical and infrared regions of the spectrum can be

written as
$$n = 1 + \frac{A(\lambda)p}{T + 273} 10^{-6}$$
 [19], where $A(\lambda)$ is a function of wavelength, p is pressure in hPa

(hectopascals), and T is temperature in Celsius. This dry air formula, which gives refractive indexes to 10^{-5} , is sufficient since differences 'due to the typical humidity of the air only amount to about 10^{-6} compared to dry air at the same atmospheric conditions' [3]. The value for $A(\lambda)$ is obtained by calculations with equations from the classic paper by Birch and Downs [20]. For $\lambda = 633$ nm, near the dominant wavelength of our red-orange sign, the index of refraction for dry air is

$$n(\lambda = 633 \text{ nm}) = 1 + \frac{78.6 p}{T + 273} 10^{-6}.$$
 (6)

Using $T_{cold} = 27.5$ °C and standard atmospheric pressure to three significant figures, p = 1010 hPa, equation (6) gives $n_{cold} = 1.00026$. The barometric pressure at the observation time was equal to standard atmospheric pressure to three significant figures.

The refraction index for $T_{hot} = 53.0$ °C is $n_{hot} = 1.00024$, giving a mirage angle from equation (5):

$$\alpha = \sqrt{2(1 - \frac{n_{\text{hot}}}{n_{\text{cold}}})} = \sqrt{2(1 - \frac{1.00024}{1.00026})} = 0.0063 \text{ rad} = 0.36^{\circ}.$$
 (7)

Before proceeding any further, let's redo the calculation for 400 nm violet light to assess dispersion errors. Then equation (6) becomes [20]

$$n(\lambda = 400 \text{ nm}) = 1 + \frac{80.4 \, p}{T + 273} 10^{-6}$$
 (8)

For the violet extreme of the spectrum equation (8) gives $n_{cold} = 1.00027$, $n_{hot} = 1.00025$, and $\alpha = 0.36^{\circ}$, leaving the mirage angle unchanged to two significant figures. The same result is interestingly obtained across the visible wavelengths to $\lambda = 700 \text{ nm}$. So dispersion is not an issue.

Using equation (6) and then equation (5), the extreme values for both the cold temperature $T_{cold} = 27.5 \text{ °C} \pm 1.0 \text{ °C}$ and the hot temperature $T_{hot} = 53.0 \text{ °C} \pm 3.0 \text{ °C}$, lead to mirage angle extremes of 0.34° and 0.39° (rounding off at the very end of the calculations). So the mirage angle from theory can be reported to two significant figures as

$$\alpha_{\rm th} = 0.36^{\circ} \pm 0.03^{\circ} \,, \tag{9}$$

which compares very well with the experimental result $\alpha_{exp} = 0.34^{\circ} \pm 0.02^{\circ}$.

Conclusion

Students will enjoy watching for mirages and taking videos of them, even if they do not pursue the theoretical analysis. They should pick a hot sunny day during the afternoon when the maximum temperature of the day occurs. In this way, the pavement temperature can best be estimated from the graphs in reference 18 which give maximum values. It might be fun to have them first guess what the angle might be. They can quickly estimate the mirage angle without a video in the following way. The height of a sign post similar to the one in this paper can be estimated as 2 m and the height of the observer sitting in the car to be 1 m above the ground. The car speed can be obtained by looking at the speedometer or asking the driver. The time to reach the sign from the vanishing mirage location can be obtained with a stop watch app on an smartphone. No video is required. In the example of this paper the distance is (31.3 m/s)(16 s) = 501 m. The proportional

horizontal length of the larger triangle in figure 3 becomes $501 \text{ m} \left[\frac{2 \text{ m}}{1 \text{ m} + 2 \text{ m}} \right] = 334 \text{ m}$. The mirage

angle is then $\alpha = \tan^{-1} \left[\frac{2}{334} \right] = 0.34^{\circ}$, the same value obtained from detailed analysis of the video.

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